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Determination of drying times for regular multi-dimensional objects

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Abstract

Drying of multi-dimensional food products is investigated analytically. A simple method is developed for the determination of drying time of multi-dimensional products using drying parameters that are available from the literature or can be determined experimentally. In this respect, geometric shape factors for different regular multi-dimensional products are introduced. Drying time that can be calculated analyticaly for an infinite slab geometry is used to predict the drying times for other multi-dimensional products by means of the geometric shape factors. The present model is verified through comparison with experimental drying times of several food products in different shapes that are obtained from the literature. The comparison of the results obtained from the present model with the experimental data shows that the predictions are accurate within ±10% range, in general. © 2002 Elsevier Science Ltd. All rights reserved.

1. Introduction

Drying of particulate objects, which is a process of simultaneous heat and mass transfer, is an energy-intensive operation of some industrial significance. It is a process whereby the moisture is vaporized and swept away from the surface, sometimes in vacuum but normally by means of a carrier fluid passing through or over the moist object. This process has found industrial applications in various forms ranging from wood drying in the lumber industry to food drying in the food industry. In the process, the heat may be added to the object from an external source, by convection, conduction, or radiation, or the heat can be generated internally within the solid body by means of electric resistance. However, regardless of the mode of heating, the moisture is always removed in a vapor phase.

There have been several experimental and theoretical studies on the determination of moisture diffusion and moisture transfer coefficients and on the analysis of heat and moisture transfer during drying of food products, undertaken by several researchers and engineers [1–14].

Zogzas and Maroulis [9] critically examined the effect of using different methods of analysis on the same experimental drying data since the moisture diffusivity is the most crucial property in drying calculations and the literature data are scarce due to the variation of both experimental measurement techniques and methods of analysis. They applied detailed and simplified mathematical models incorporating moisture diffusivity as a model parameter and found that significant differences in the calculated values of moisture diffusivity result when different models are employed. They also conducted experimental work using a typical pilot plant dryer with controlled air conditions and proved that the moisture content dependence of the diffusion coefficient is of significance at the last drying stage based on the well-known Arrhenius relation. Furthermore, the effects of considering external mass transfer and volume shrinkage during drying were investigated. The authors made a significant contribution with their work to the current drying literature and provide some practical tips to the drying industry.

Dincer and Dost [1,11] first developed new analytical models in a simple and accurate manner to determine the moisture diffusivity and moisture transfer coefficients

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Nomenclature

a	thermal diffusivity (m^2/s)
Bi	Biot number ($= h_m L/D$)
D	moisture diffusivity (m^2/s)
E	shape factor
F	heat transfer surface area (m^2)
Fo	Fourier number ($= Dt/L^2$)
h_m	mass transfer coefficient (m/s)
G	lag factor
L	characteristic dimension (m)
S	drying coefficient (1/s)
t	time (s)
T	temperature (K)
U	velocity (m/s)
V	volume (m^3)
W	moisture content (kg/kg)
y	coordinate (m)
z	coordinate (m)
β_1	ratio of second dimension to the characteristic length
β_2	ratio of third dimension to the characteristic length
σ	characteristic dimension (m)
Φ	dimensionless moisture content
θ	dimensionless temperature

Subscripts

a	ambient
c	center
e	equilibrium
i	initial

for the geometrically shaped products subject to a drying process, by using the moisture profiles within the products. They also introduced new drying parameters in terms of drying coefficient and lag factors, based on a similarity occurring between the cooling and drying profiles since these profiles decrease in an exponential form with process time. Therefore, they also first incorporated these new drying parameters into the analytical models developed. The developed moisture diffusivity and moisture transfer coefficient models were verified with the actual data sets and remarkably good agreement was found between the model results and experimental data. But there is a need to do further research in this topic, particularly for the food products with irregular shapes. In this regard, it will be quite useful to develop some shape factors and incorporate these into the models. In addition, Dincer [15] developed a new dimensionless number for forced-convection heat transfer during heating and cooling of solid objects, expressing the effect of flow velocity of heating or cooling fluid (as the fluid property) on the heating and cooling coefficient (as the thermal parameter) of the object with regular and irregular shapes. Now, it is intended to extent this to food drying applications.

In this regard, an accurate analysis of moisture transfer during drying of multi-dimensional objects is essential. Although there are many moisture transfer models (e.g., Arrhenius equation, Fick's law for diffusion, the lumped model) available in the literature to study the one-dimensional moisture diffusion and to determine drying times, moisture content distributions and moisture transfer parameters, e.g., moisture diffusivities and moisture transfer coefficients [2–14,16], to the best of the authors' knowledge no model is available for multi-dimensional shapes for the solid objects subject to drying process in the open literature. It is important to note that in the literature, there are some models available for mass transfer from the liquids, not from the solids. It is also important to mention that at the present time we are able to use some advanced computational techniques for such processes to obtain whatever parameter is needed. However, these require the computational softwares and workstations (or PCs) allocated for. In drying industry, practitioners need simple, but accurate analytical tool to conduct the design analysis and relevant calculations. This study is undertaken analytically to develop such models for practical applications.

In the present paper, a simple model of moisture transfer for multi-dimensional products is presented. Making use of the analogy between heat conduction and moisture transfer, drying time for infinite slab products is formulated. The analysis is then extended to multi-dimensional products through the geometric shape factors introduced. An illustrative example is presented to highlight the importance of the present model and show how to use the present methodology to determine shape factors, along with other relevant parameters such the Biot number and dimension ratios.

2. Analysis of heat and moisture transfer

2.1. Modeling drying process of infinite solid slab products

The transient moisture diffusion process, which occurs during drying of solid objects, is similar in form to the process of heat conduction in these objects. The governing Fickian equation is exactly in the form of the Fourier equation of heat transfer, in which temperature and thermal diffusivity are replaced with concentration and moisture diffusivity, respectively. Therefore, similar to the case of unsteady heat transfer, one can consider three different situations for the unsteady moisture diffusion, namely, the cases where the Biot number has the following values: $Bi \leqslant 0.1$, $0.1 < Bi < 100$, and $Bi > 100$. The first case, corresponding to situations where $Bi \leqslant 0.1$, imply negligible internal resistance to the moisture diffusivity within the solid object. On the other hand, cases where $Bi > 100$, including negligible surface resistance to the moisture transfer at the solid object, are the most common situation, while cases where $0.1 < Bi < 100$, including the finite internal and surface resistances to the moisture transfer, exist in practical applications.

The time-dependent heat and moisture transfer equations in Cartesian, cylindrical, and spherical coordinates for an infinite slab, infinite cylinder, and a sphere, respectively, can be written in the following compact form:

$$\left(\frac{1}{y^m}\right)\left(\frac{\partial}{\partial y}\right)\left[y^m\left(\frac{\partial T}{\partial y}\right)\right] = \left(\frac{1}{a}\right)\left(\frac{\partial T}{\partial t}\right) \quad \text{for heat transfer} \tag{1}$$

and

$$\left(\frac{1}{y^m}\right)\left(\frac{\partial}{\partial y}\right)\left[y^m\left(\frac{\partial W}{\partial y}\right)\right] = \left(\frac{1}{D}\right)\left(\frac{\partial W}{\partial t}\right) \quad \text{for moisture transfer,} \tag{2}$$

where $m = 0$, 1, and 2 for an infinite slab, infinite cylinder, and a sphere. $y = z$ for an infinite slab, $y = r$ for infinite cylinder and sphere. T represents temperature (°C), W is moisture content by weight as dry basis (kg/kg), a is thermal diffusivity (m^2/s), D is moisture diffusivity (m^2/s), and t is time (s).

Using the experimental data in the mathematical model developed, the dimensionless temperature (θ) and dimensionless moisture content (Φ) can be defined as follows:

$$\theta = (T - T_i)/(T_a - T_i), \tag{3}$$

$$\Phi = (W - W_e)/(W_i - W_e), \tag{4}$$

where subscripts a, e, and i indicate ambient, equilibrium, and initial conditions, respectively.

The following assumptions are made:

- Thermophysical properties of the solid and the drying medium are constant.
- The effect of heat transfer on the moisture loss is negligible.
- The moisture diffusion occurs in the z direction (perpendicular to the slab surface) only.

Under these conditions, the governing one-dimensional moisture diffusion equation, Eq. (2), can be written as

$$D\frac{\partial^2 \Phi}{\partial z^2} = \frac{\partial \Phi}{\partial t}. \tag{5}$$

The following initial and boundary conditions are considered:

$$\Phi(z, 0) = 1, \tag{6}$$

$$(\partial \Phi(0, t)/\partial z) = 0, \tag{7}$$

$$-D(\partial \Phi(L, t)/\partial z) = h_m \Phi(L, t) \quad \text{for } 0.1 \leqslant Bi \leqslant 100, \tag{8}$$

$$\Phi(L, t) = 0 \quad \text{for } Bi > 100,$$

and

$$-h_m F \Phi(L, t) = \rho V c_p \partial(\Phi(L, t))/\partial t \quad \text{for } Bi < 0.1, \tag{9}$$

where F is heat transfer surface area, L is half thickness of slab and the Biot number is $Bi = h_m L/D$.

The solution of Eq. (5) is extensively treated for the regular shaped objects earlier [17,18]. So, the dimensionless average moisture distribution becomes

$$\Phi = \exp(-h_m t/\sigma) \quad \text{for the case where } Bi < 0.1 \ [\sigma = (V/F) = L] \tag{10}$$

and

$$\Phi = \sum_{n=1}^{\infty} A_n B_n \quad \text{for the case where } Bi > 0.1. \tag{11}$$

Eq. (11) can be simplified by ignoring the values of ($\mu^2 Fo$) smaller than 1.2 (taking only the first term into consideration), resulting in

$$\Phi = A_1 B_1, \tag{12}$$

where A_1 is given by

$$A_1 = 2 \sin \mu_1/(\mu_1 + \sin \mu_1 \cos \mu_1) \quad \text{for } 0.1 \leqslant Bi \leqslant 100 \tag{13}$$

and

$$A_1 = 2/\mu_1 \quad \text{for } Bi > 100, \tag{14}$$

and B_1 is

$$B_1 = \exp\left(-\mu_1^2 Fo\right) \quad \text{for } Bi > 0.1, \tag{15}$$

where the Fourier number is defined as $Fo = Dt/L^2$.

The corresponding characteristic equations are given as

$$\cot(\mu_1) = (1/Bi)\mu_1 \quad \text{for } 0.1 \leqslant Bi \leqslant 100 \tag{16}$$

and

$$\mu_1 = \pi/2 \quad \text{for } Bi > 100. \tag{17}$$

Due to the fact that drying has an exponentially decreasing trend, we establish the following equation for the objects subject to drying, by introducing lag factor (G, dimensionless) and drying coefficient (S, 1/s):

$$\Phi = G \exp(-St). \tag{18}$$

Here drying coefficient shows the drying capability of the object or product and lag factor is an indication of internal resistances of object to the heat and/or moisture transfer during drying. Both Eqs. (12) and (18) are in the same form and can be equated to each other by having $G = A_1$ and $S = \mu_1^2 D/L^2$. Therefore:

Moisture transfer coefficient:

$$h_m = \sigma S \quad \text{for } Bi < 0.1, \tag{19}$$

$$h_m = (D/L)Bi \quad \text{for } Bi > 0.1. \tag{20}$$

Moisture diffusivity:

$$D = SL^2/\mu_1^2, \tag{21}$$

where μ_1 is given in Eq. (16).

2.2. Drying time for infinite solid slab product

Analytical drying time of a slab object can be obtained using Eqs. (10) and (12) as

$$t_{slab} = -\frac{\sigma}{h_m} \ln \Phi_c \quad \text{for } Bi < 0.1, \tag{22}$$

$$t_{slab} = -\frac{L^2}{D\mu_1^2} \ln\left(\frac{\Phi_c}{A_1}\right) \quad \text{for } Bi \geqslant 0.1, \tag{23}$$

where Φ_c is the dimensionless centerline moisture content.

On the other hand, using the experimental model given in Eq. (18), the drying time can be related to the lag factor and drying coefficient as

$$t_{slab} = -\frac{1}{S} \ln\left(\frac{\Phi_c}{G}\right) \quad \text{for } 0.1 \leqslant Bi < 100. \tag{24}$$

3. Drying time for regular multi-dimensional shaped products

In practice, various types of objects (foods, woods, etc.) that are to be dried are usually of three-dimensional shape. Therefore, accurate determination of drying time for multi-dimensional products becomes an important issue, since this directly relates to conservation of energy and quality of the product. Due to the complexity of the problem, an analytical solution may not be possible. However, introducing a geometric shape factor, a reasonably accurate drying time for regular multi-dimensional shaped object, t_{shape}, can be calculated as

$$E = \frac{t_{slab}}{t_{shape}}, \tag{25}$$

where E is the geometric shape factor that is a function of the shape and the Biot number. The analytical expressions for the geometric shape factor are proposed by Hossain et al. [19] for cooling applications.

4. Geometric shape factors

In all the geometrical shapes considered, L is the characteristic dimension (smallest distance from center to surface) and the parameters β_1 and β_2 are the ratios of the second and third dimensions to the characteristic dimension, respectively.

4.1. Infinite rectangular rod of sides $2L \times 2\beta_1 L$

When the axial dimension of a rectangular shape product is relatively large, the product can be considered to be an infinite rectangular rod. This means that the end effects of the axial direction are neglected and the diffusion process is considered to be two-dimensional. This will be a good approximation when the axial dimension is larger than 4–5 times the second largest dimension [20]. In this case, the geometric shape factor is obtained as follows [20,21]:

$$E = \left(1 + \frac{2}{Bi}\right)\left\{\left(1 + \frac{2}{Bi}\right) - 4\sum_{n=1}^{\infty}\left[\frac{\sin z_n}{z_n^3\left(1 + \frac{\sin^2 z_n}{Bi}\right)\left(\frac{z_n}{Bi}\sinh(z_n\beta_1) + \cosh(z_n\beta_1)\right)}\right]\right\}^{-1}, \tag{26}$$

where the values of z_n are the roots of

$$Bi = z_n \tan z_n. \tag{27}$$

Considering only the first term in Eq. (26), the shape factor can be approximated as

$$E = \left(1 + \frac{2}{Bi}\right)\left\{\left(1 + \frac{2}{Bi}\right) - \frac{4 \sin z_1}{z_1^3\left(1 + \frac{\sin^2 z_1}{Bi}\right)\left(\frac{z_1}{Bi}\sinh(z_1\beta_1) + \cosh(z_1\beta_1)\right)}\right\}^{-1}, \tag{28}$$

where the values of z_1 are the roots of

$$Bi = z_1 \tan z_1. \tag{29}$$

4.2. Rectangular brick of dimensions $2L \times 2\beta_1 L \times 2\beta_2 L$

Some of the pre-processed food products such as meat cuts, poultry, etc., may be approximated as rectangular brick. In this case, the diffusion process is three-dimensional. The geometric shape factor for this geometry is obtained as follows:

$$E = \left(1+\frac{2}{Bi}\right)\left\{\left(1+\frac{2}{Bi}\right) - 4\sum_{n=1}^{\infty}\left[\frac{\sin z_n}{z_n^3\left(1+\frac{\sin^2 z_n}{Bi}\right)\left(\frac{z_n}{Bi}\sinh(z_n\beta_1)+\cosh(z_n\beta_1)\right)}\right] - 8\beta_2^2\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\sin z_n \sin z_m \times \left\{\left[\cosh(z_{nm}) + \frac{z_{nm}}{Bi\beta_2}\sinh(z_{nm})\right] z_n z_m z_{nm}^2\left(1+\frac{1}{Bi}\sin^2 z_n\right) \times \left(1+\frac{1}{Bi\beta_1}\sin^2 z_m\right)\right\}^{-1}\right]\right\}^{-1}, \tag{30}$$

where the values of z_n and z_m are the roots of

$$Bi = z_n \tan z_n \tag{31}$$

and

$$(Bi \times \beta_1) = z_m \tan z_m, \tag{32}$$

respectively, and the values of z_{nm} are given by

$$z_{nm}^2 = z_n^2\beta_2^2 + z_m^2\left(\frac{\beta_2}{\beta_1}\right)^2. \tag{33}$$

In order to obtain a simpler and easy-to-use formula, consider only the first term in Eq. (28). The accuracy with this assumption depends on the dimensions of the product. When the three dimensions of the product are nearly the same size, more terms should be included for better accuracy. The geometric shape factor, in this case, can be approximated as

$$E = \left(1+\frac{2}{Bi}\right)\left\{\left(1+\frac{2}{Bi}\right) - \frac{4\sin z_1}{z_1^3\left(1+\frac{\sin^2 z_1}{Bi}\right)\left(\frac{z_1}{Bi}\sinh(z_1\beta_1)+\cosh(z_1\beta_1)\right)} - 8\beta_2^2\left[\sin z_1 \sin z_2 \times \left\{\left[\cosh(z_{12}) + \frac{z_{12}}{Bi\beta_2}\sinh(z_{12})\right] z_1 z_2 z_{12}^2\left(1+\frac{1}{Bi}\sin^2 z_1\right) \times \left(1+\frac{1}{Bi\beta_1}\sin^2 z_2\right)\right\}^{-1}\right]\right\}^{-1}, \tag{34}$$

where the values of z_1 and z_2 are the roots of

$$Bi = z_1 \tan z_1 \tag{35}$$

and

$$(Bi \times \beta_1) = z_2 \tan z_2, \tag{36}$$

respectively, and the values of z_{nm} are given by

$$z_{12}^2 = z_1^2\beta_2^2 + z_2^2\left(\frac{\beta_2}{\beta_1}\right)^2. \tag{37}$$

4.3. Finite cylinder of radius L and height $2\beta_1 L$ (height exceeds diameter)

Many food products (e.g., cucumbers, eggplants, zucchini) are of cylindrical shape. For the products with their axial dimension (height) larger than the diameter, the geometric shape factor is obtained as follows [20,21]:

$$E = \left(2+\frac{4}{Bi}\right) \times \left\{\left(1+\frac{2}{Bi}\right) - 8\sum_{n=1}^{\infty}\left[y_n^3 J_1(y_n)\left(1+\frac{y_n^2}{Bi^2}\right) \times \left(\cosh(\beta_1 y_n) + \frac{y_n}{Bi}\sinh(\beta_1 y_n)\right)\right]^{-1}\right\}^{-1}, \tag{38}$$

where the values of y_n are the roots of

$$y_n J_1(y_n) - Bi J_0(y_n) = 0 \tag{39}$$

and J_0 and J_1 are the Bessel functions of first kind of order 0 and 1, respectively:

$$J_0(y) = 1 - \frac{y^2}{2^2} + \frac{y^4}{2^2 4^2} - \frac{y^6}{2^2 4^2 6^2} + \cdots, \tag{40}$$

$$J_1(y) = \frac{y}{2} - \frac{y^3}{2^2 4} + \frac{y^5}{2^2 4^2 6} - \frac{y^7}{2^2 4^2 6^2 8} + \cdots \tag{41}$$

For the same reasons stated above and considering only the first term in Eq. (38), the shape factor can be approximated as

$$E = \left(2+\frac{4}{Bi}\right) \times \left\{\left(1+\frac{2}{Bi}\right) - 8\left[y_1^3 J_1(y_1)\left(1+\frac{y_1^2}{Bi^2}\right) \times \left(\cosh(\beta_1 y_1) + \frac{y_1}{Bi}\sinh(\beta_1 y_1)\right)\right]^{-1}\right\}^{-1}, \tag{42}$$

where the values of y_1 are the roots of

$$y_1 J_1(y_1) - Bi J_0(y_1) = 0. \tag{43}$$

4.4. Finite cylinder of radius $\beta_1 L$ and height 2L (diameter exceeds height)

For the products that are circular and with their height less than the diameter (e.g., peaches, meat burgers, cheese, potato cuts), the geometric shape factor is obtained as follows:

$$E = \left(1+\frac{2}{Bi}\right) \times \left\{\left(1+\frac{2}{Bi}\right) - 4\sum_{n=1}^{\infty}\left[\frac{\sin z_n}{z_n^2[z_n + \cos z_n \sin z_n]\left(I_0(z_n\beta_1) + \frac{z_n}{Bi}I_1(z_n\beta_1)\right)}\right]\right\}^{-1}, \tag{44}$$

where the values of z_n are the roots of

$$Bi = z_n \tan z_n \tag{45}$$

and I_0 and I_1 are the Bessel functions of second kind of order 0 and 1, respectively:

$$I_0(z) = 1 + \frac{z^2}{2^2} + \frac{z^4}{2^2 4^2} + \frac{z^6}{2^2 4^2 6^2} + \cdots, \tag{46}$$

$$I_1(z) = \frac{z}{2} + \frac{z^3}{2^2 4} + \frac{z^5}{2^2 4^2 6} + \frac{z^7}{2^2 4^2 6^2 8} + \cdots. \tag{47}$$

Again, considering only the first term in Eq. (44), the shape factor can be approximated as

$$E = \left(1 + \frac{2}{Bi}\right) \times \left\{ \left(1 + \frac{2}{Bi}\right) - \frac{4 \sin z_1}{z_1^2 [z_1 + \cos z_1 \sin z_1] \left(I_0(z_1 \beta_1) + \frac{z_1}{Bi} I_1(z_1 \beta_1)\right)} \right\}^{-1}, \tag{48}$$

where the values of z_1 are the roots of

$$Bi = z_1 \tan z_1. \tag{49}$$

4.5. Infinite elliptical cylinder of semi-minor axis L and semi-major axis $\beta_1 L$

Long axial food products with elliptical cross-section, a simple and reasonably accurate geometric shape factor is obtained as follows:

$$E = 1 + \frac{1 + \frac{2}{Bi}}{\beta_1^2 + \frac{2\beta_1}{Bi}}. \tag{50}$$

4.6. Ellipsoid having semi-axes of $L, \beta_1 L$, and $\beta_2 L$

When the axial dimension is finite, then the diffusion process becomes three-dimensional. In general, all the irregular shape products can be approximated to be ellipsoid. In this case, a simple form of geometric shape factor is written as

$$E = 1 + \frac{1 + \frac{2}{Bi}}{\beta_1^2 + \frac{2\beta_1}{Bi}} + \frac{1 + \frac{2}{Bi}}{\beta_2^2 + \frac{2\beta_2}{Bi}}. \tag{51}$$

Two special cases can be of common interest. Many food products appear to have these two shapes. They are as follows:

(a) *Prolate spheroid*, $\beta_1 = 1 \Rightarrow$ egg model:

$$E = 2 + \frac{1 + \frac{2}{Bi}}{\beta_2^2 + \frac{2\beta_2}{Bi}}. \tag{52}$$

Oblate spheroid, $\beta_1 = \beta_2 = \beta \Rightarrow$ hamburger model:

$$E = 1 + 2\frac{1 + \frac{2}{Bi}}{\beta^2 + \frac{2\beta}{Bi}}. \tag{53}$$

5. Results and discussion

In order to demonstrate the accuracy and usefulness of the above given geometric shape factors in calculating the drying times, first the effects of drying and geometric parameters such as Biot number (Bi), β_1 and β_2 are discussed. Here the Biot number for drying shows the magnitude of the internal and external resistances to moisture transfer from the product. The shape factor shows the ratio of the drying time of a regular slab object to the drying time of a multi-dimensional object. β_1 and β_2 are representing the ratios of second and third dimensions to the characteristic lengths. Then the predicted drying times obtained using these geometric shape factors can be compared with the experimental drying results obtained from the literature.

Fig. 1 shows the drying shape factor (E) with Biot number (Bi) as β_1 is variable for infinite rectangular rod. E attains high values at $Bi \leqslant 0.1$ and as Bi increases E reduces rapidly for all β_1 values. This behavior can also be observed from Eqs. (26)–(29), i.e., $Bi > 0.16$ the effect of Bi almost diminishes and E becomes almost β_1 dependent. In this case, it is the geometric configuration of the object which defines the E, i.e., the geometric configuration effects the drying time ratio (t_{slab}/t_{shape}). The effect of β_1 on E is significant. Increasing β_1 reduces the value of E. This indicates that increasing β_1, defining the size of the object, increases the size of the object which in turn results in lower drying time for a small size slab than the slab corresponding to the large size.

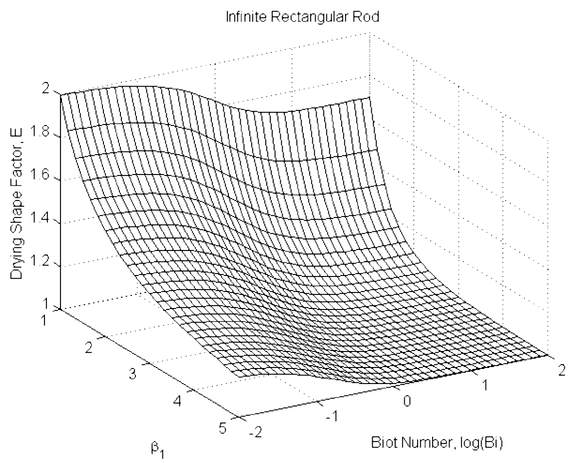

Fig. 1. Drying shape factor (E) versus Biot number for various geometric factors β_1 for infinite rectangular rod.

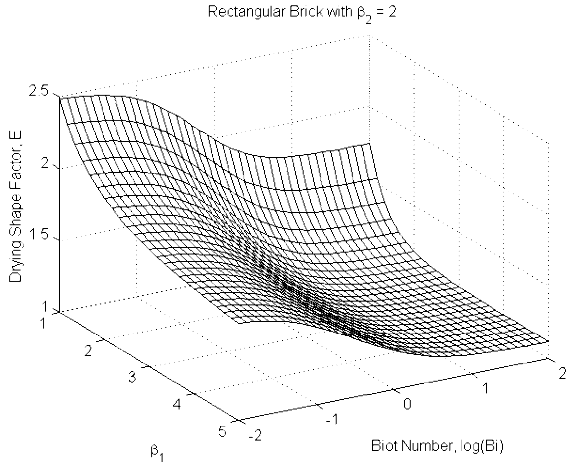

Fig. 2. Drying shape factor (E) versus Biot number for various geometric factors β_1 for rectangular brick with $\beta_2 = 2$.

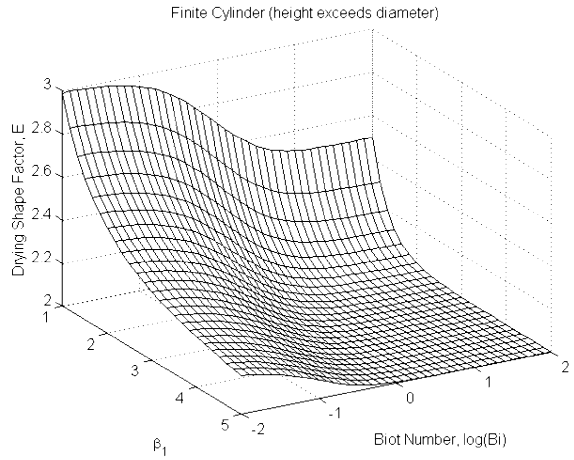

Fig. 3. Drying shape factor (E) versus Biot number for various geometric factors β_1 for finite cylinder (height exceeds diameter).

Fig. 2 shows the drying shape factor (E) with Biot number (Bi) as β_1 is variable for rectangular brick. E reduces with increasing Bi as well as with increasing β_1. This behavior is also true for all β_2 values employed in the analysis. The decay of E with Bi is more pronounced for $Bi \geqslant 0.1$. In this case, increasing Bi results in increasing moisture transfer coefficient, which lowers the drying shape factor. Moreover, the effect of β_1 on the drying shape factor is significant for all Biot numbers. This shows that enlarging the size of the brick reduces the drying shape factor, which is true for all Biot numbers. The effect of third dimension ratio (β_2) on the drying shape factor is becoming identical to that of β_1. As shown in Fig. 2, increasing β_2 reduces the value of drying shape factor. Moreover, the behavior of drying shape factor with Biot number does not change by changing β_2, due to the 0fact that the Biot number is a function of the moisture transfer coefficient, the characteristic dimension and the moisture diffusivity. Furthermore, both the Biot number and the shape factor are related to the characteristic dimension.

Figs. 3–6 show drying shape factor with Biot number (Bi) as β_1 is variable for finite cylinder, elliptic cylinder, and ellipsoid. In general, all the cases, drying shape factor reduces with increasing Biot number. The behavior of curves is similar to those corresponding to the previous cases (Figs. 1 and 2). The effect of size of the cylinder on the drying shape factor is insignificant as seen from Figs. 3 and 4. In the case of elliptic cylinder, the value of drying shape factor reduces and the variation of drying shape factor with Biot number is not substantial as compared to that for circular cylinders. This is more apparently for $\beta_1 = 1$. This argument is also true for ellipsoid for $\beta_2 = 1$ as shown in Fig. 6. The effect of β_2 on the drying shape factor is identical to that

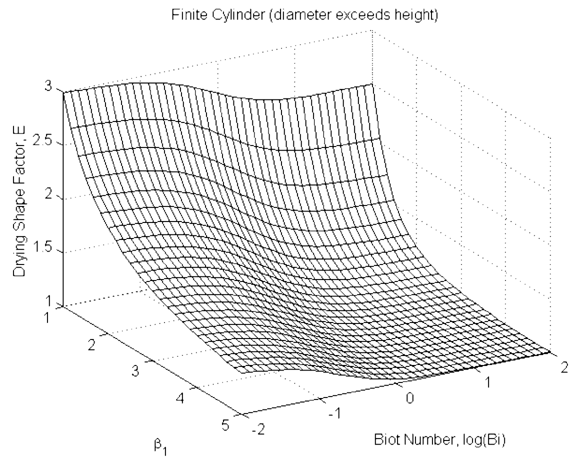

Fig. 4. Drying shape factor (E) versus Biot number for various geometric factors β_1 for finite cylinder (diameter exceeds height).

of β_1. In this regard, as β_2 increases, the variation of drying shape factor with Biot number becomes considerable, provided that the value of drying shape factor reduces as β_2 increases.

5.1. Model verification and illustrative example

In order to verify the accuracy of the shape factors given above, several products in different shapes are selected from the literature as given in Table 1. The characteristic length (L), diffusion coefficient (D), moisture transfer coefficient (h_m) and the experimental drying time (t_{exp}) for corresponding centerline moisture content (Φ_c) are based on experimental values and are obtained

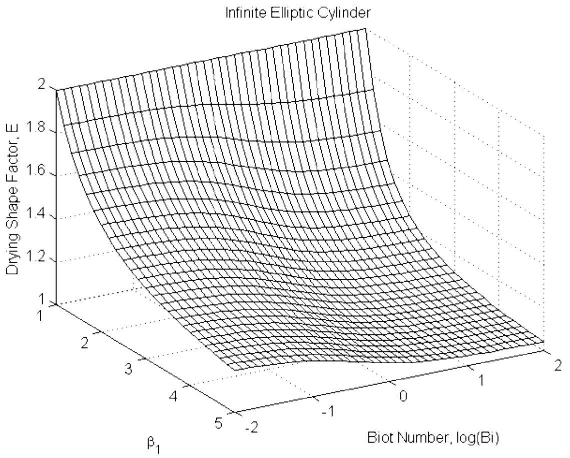

Fig. 5. Drying shape factor (E) versus Biot number for various geometric factors β_1 for infinite elliptic cylinder.

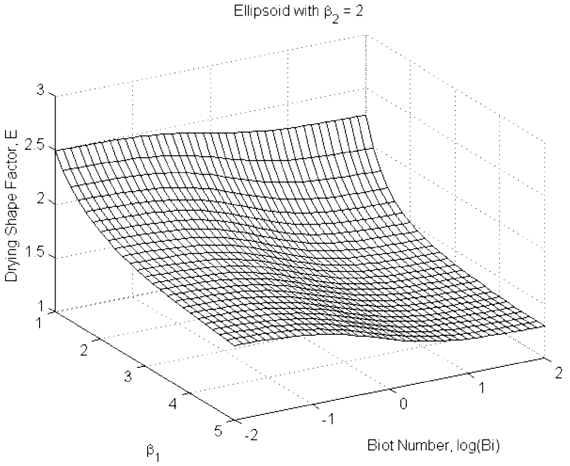

Fig. 6. Drying shape factor (E) versus Biot number for various geometric factors β_1 for ellipsoid with $\beta_2 = 2$.

as outlined in the previous papers [18,22]. The predicted theoretical drying time (t_{pred}) for these products have been calculated using the present method and are also given in Table 1 next to the experimental values (t_{exp}) for comparison. The agreement between the experimental and theoretical drying times has been shown in the last column of Table 1. The agreement is good in general, except for the case of okra, for which a deviation of 11.5% is obtained between the experimental measurement and theoretical prediction. This may be due to some measurement errors involved and the assumptions made for the shape of the product in the model. The assumptions that might have resulted in some discrepancies between the actual and the model results are the simplification in considering the first term only in the

Table 1
Experimental data and theoretical predictions of drying times for several products, along with experimental conditions (e.g., drying air temperature T_a, relative humidity of air RH, air velocity U)

Product	Shape	T_a (°C)	RH (%)	U (m/s)	L (m)	$D = (SL^2)/\mu$ (m^2/s)	$h_m = (Bi \cdot D)/L$ (m/s)	Φ_c	t_{exp} (s)	t_{pred} (s)	Percentage of error
Carrot [23]	Slab	50	–	2.5	0.005	5.189×10^{-09}	6.6084×10^{-07}	0.2	16 200	15713.1	−3
Prune [24]	Slab	60	15	3–5	0.005	3.854×10^{-08}	4.0261×10^{-07}	0.1	28 800	28595.9	−0.7
Okra [25]	Cylinder	60	6–12	1.2	0.003	8.638×10^{-10}	2.8219×10^{-07}	0.03	46 800	52158.9	11.5
Starch powder [26]	Cylinder	62	–	2	0.005	1.294×10^{-07}	3.1216×10^{-06}	0.37	1800	1693.63	−5.9
Grape [27]	Sphere	50	–	0.5	0.0009	3.731×10^{-10}	1.0541×10^{-07}	0.05	28 800	28580.3	−0.76
Potato [28]	Sphere	40	–	1	0.009	9.42×10^{-07}	8.3156×10^{-06}	0.33	1200	1199.9	0
Thompson seedless grape [29]	Sphere	50	15	0.25–1	0.0009	3.919×10^{-11}	1.2696×10^{-07}	0.03	54 000	54846.8	1.57

solution, the uniformity of the product and the constant moisture diffusivity and moisture transfer coefficient.

6. Conclusion

Drying of multi-dimensional shape food products has been analyzed. Considering the analogy between the heat diffusion and moisture transfer, a simple model for the drying time of solid food products has been developed. Previously derived geometric shape factors have been introduced to evaluate drying times for multi-dimensional objects. In general, one-term approximation is found to be sufficient. The analytical predictions using the one-term approximation showed that the accuracy of the present simple model remains within $\pm 10\%$ range. However, the accuracy may be improved by considering two or more terms in the analysis. Future work will be conducted to develop shape factors for irregular shaped products and to study the effect of shrinkage on the moisture diffusion.

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